

$$Q1. \lim_{x \rightarrow 1^+} \frac{x}{x-1} = +\infty$$

State a computation rule to enable you to get the above conclusion.

Q2*. Check Q1 in the ϵ - δ terminology.

Hint(想). Let $M \in \mathbb{R}$. Wish to find $\delta > 0$ s.t.

$$(*) \quad \frac{x}{x-1} > M \text{ whenever } x \in (1, 1+\delta)$$

$$? \quad x > M(x-1) = Mx - M$$

$$\Leftrightarrow M > (M-1)x$$

$$\Leftrightarrow \frac{M}{M-1} > x \quad (\text{Assume } M > 1)$$

$$\parallel$$
$$\frac{M-1+1}{M-1}$$

$$\Leftrightarrow 1 + \frac{1}{M-1} > x$$

Thus one may take $\delta := \frac{1}{M-1}$
(with $M > 1$).

Q3* Do Q1, Q2 but for

$$\lim_{x \rightarrow 1^-} \frac{x}{x-1} = -\infty$$

Q4 (= q. 9 of § 4.3). Hint:

$$|f(x)| = \frac{|x f(x)|}{|x|} \quad \forall x \neq 0$$

Q5 (= q. 5 (a), (b*), (c), (d), (e*), (g), (h))

Q6 (= q. 11, 12, 13 of § 4.3, Bartle)